THE MATHEMATICAL GRAMMAR SCHOOL CUP

- MATHEMATICS -

June 2024.

1. If the pair (x, y) is a solution of the system of equations $2-x^3 = y, 2-y^3 = x$, then sum of the coordinates of all pairs is:

(A) $+\infty$ (B) -11 (C) 2 (D) $2+\sqrt{2}$ (E) 4.

2. On the circumference of a circle are n distinct real numbers, $n \geqslant 3$, in such a way that each number is equal to the product of its immediate neighbors. The sum of squares of all such numbers n are:

(A) 9 (B) 136 (C) 45 (D) 244 (E) 36.

3. Let $\triangle ABC$ be a triangle with sides $|AB| = 21$, $|AC| = 28$, $|BC| = 35$. Let D, E, F be the feet of the angle bisectors from A, B and C, respectively. The area of the triangle $\triangle DEF$ is:

(A) 70 (B) 75 (C) 80 (D) 85 (E) 90.

4. Let c be a real number. If the set of points in the xy-plane which satisfy the equation

$$x^4 - cy^2 + (1-c)x^2 - 2x^3 + x^2y^2 + 2cx - c = 0$$

contains the vertices of exactly one equilateral triangle, then the value of c is:

(A) -2 (B) 0 (C) 2 (D) 4 (E) 6.

5. You are given 19 rods, of lengths 1, 2, 3, ..., 19 (one rod of each length). The number of ways you can choose three different rods such that they can form a triangle is:

(A) 465 (B) 423 (C) 410 (D) 492 (E) 444

6. There are 101 people seated around a circular table. It is known that one of them is 37 years old and that every person's age is the arithmetic mean of ages of a couple of people (possibly one) seated directly to the left of him. The maximal age of the oldest person at the table under the given conditions is:

(A) 74 (B) 37 (C) 69 (D) 101 (E) 64.

7. The total number of all numbers with 7 digits formed by all permutations of the digits of the number 1234567 and which are divisible by 7 is:

(A) 640 (B) 680 (C) 700 (D) 720 (E) 740.

8. Let a, b and c denote the three roots of a polynomial $Q(x) = x^3-3x-1$. If $P \in \mathbb{Z}[x]$ is a monic polynomial (with leading coefficient equal to one) of degree 6 with integer coefficients such that $P(a+\sqrt{3}) = 0$, $P(b+\sqrt{3}) = 0$ and $P(c+\sqrt{3}) = 0$, then the value of $P(1)$ is:

(A) 31 (B) 73 (C) 66 (D) 27 (E) 42.

GOOD LUCK!!!