Final Answer Table

Question	Answer	Question	Answer
01.	C	17.	B
02.	D	18.	C
03.	D	19.	D
04.	A	20.	A
05.	C	21.	D
06.	A	22.	D
07.	A	23.	B
08.	B	24.	C
09.	D	25.	B
10.	B	26.	B
11.	B	27.	E
12.	E	28.	A
13.	B	29.	B
14.	A	30.	E
15.	C	31.	E
16.	C	32.	B

Answers with Hints

Answer **1.** (C) Michael Faraday

Hint. Unlike the others, Michael Faraday had certainly nothing to do with the foundation of gravitational physics, neither had he contributed to its development.

Answer **2.** (D) $-273.15°$ C.

Answer **3.** (D) either smaller than or equal to the distance travelled.

Answer **4.** (A) Both ships have the same velocity at some instant before τ.

Answer **5.** (C) $1\ \Omega$.

Answer **6.** (A) Pascal's law.

Answer **7.** (A)

Hint. If the currents are flowing in the same direction then the magnetic induction lines around the pair of wires tend to get amassed. However, if the currents are in opposite directions, then most of the associated magnetic induction lines tend to cancel each other.

Answer **8.** (B) behind the mirror's surface.

Answer **9.** (D) 5.0 cm/s to the left.

Hint. Enter into the reference frame of point P and observe the relative speeds of both ends. Given the chain is inextensible, the movements of the ends are made in the opposite directions, but at equal relative speeds: $v_1^{(\mathrm{rel})}$ and $v_2^{(\mathrm{rel})}$. Thus, $v_1^{(\mathrm{rel})} \equiv v_1 - v_P$ must be equal to $v_2^{(\mathrm{rel})} \equiv v_P - v_2$, so that finally $v_P = (v_1 + v_2)/2$.

Answer **10.** (B) In the sense of Earth's gravity direction, it is unreliable by the randomly rotated photo to describe to an extraterrestrial alien what is *up* versus *down*.

Answer **11.** (B)

Answer **12.** (E) $E_D > E_B > E_A > E_C$

Hint. The electric field is zero inside the conductor. The electric fields are strongest at locations along the edges where the object is most curved. The curvature can range from absolute flatness with the weak electric field extreme to being curved to a sharp point with the strong electric field extreme.

Answer **13.** (B) directed rightward.

Answer **14.** (A) Stokes' law

Hint. Unlike the non-magnetic ball, the falling magnet induces an electromotive force inside the pipe walls (Faraday's law of induction). The electric resistance of copper will determine the strength of the electric current (Ohm's law) flowing inside the walls, which will eventually lead to heat generation inside the walls (Joule-Lenz law). The electric current will further create an induced magnetic field to oppose the falling magnet (Lenz's law), due to which the magnet falls increasingly slowly till a terminal speed is reached. Though the opposing magnetic force acts as a drag directed upward, this has nothing to do with Stoke's law which is concerned with fluid (air) resistance.

Answer **15.** (C) $f < s < 2f$.

Hint. The image being real requires $s > f$. Furthermore, the image being taller than the object requires $s < 2f$.

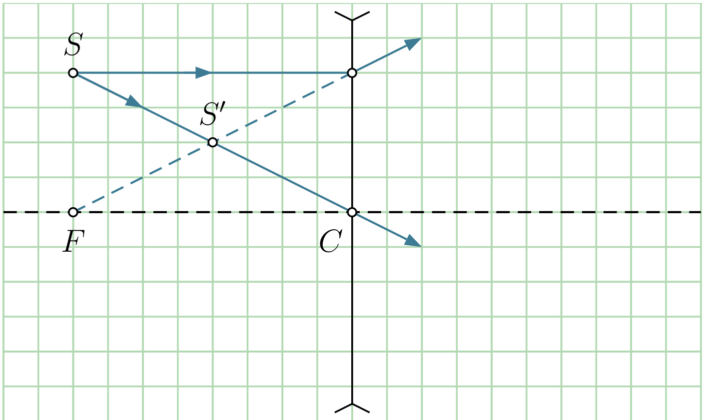
Answer 16. (C)

Hint. Apply Kirchhoff's first rule at the nodes.

Answer 17. (B) 10 N.

Answer 18. (C) $f = 8.0$ cm

Hint. Let us start with drawing a straight line SS' that passes through both object and its image. The intersection of this line with the optical axis of the lens will give us the position of the lens center C, as well as the lens itself. Then we analyze another ray that initially propagates in parallel with the optical axis. Through the extrapolation, this will give us the focal position F, and by counting the unit-cells, $|CF| = 8.0$ cm.



Answer 19. (D)

Answer 20. (A) $\varphi_2/\varphi_1 = 3/2$.

Hint. The electric potential at the tip of the pyramid is proportional to its total charge q divided by the characteristic length a. As $\rho = \mathrm{const}$, then $q \propto a^3$, so that $\varphi_1 \propto a^2$. If $a \to a/2$ then $\varphi_1 \to \varphi_1/4$. There are exactly six such pyramids inside the cube of dimensions $a \times a \times a$, so that $\varphi_2 = 6 \times \varphi_1/4$ by superposition principle.

Answer 21. (D) $\tau_C > \tau_B > \tau_A$

Hint. The time to impact only depends upon the vertical projection of the initial speed, which in turn determines the maximum height. The higher the maximum height, the longer the trajectory time takes.

Answer 22. (D) $\mu = \sqrt{3}/6$

Hint. There is a $3:1$ ratio between the accelerations (up/down). This ratio can be explained by the change in the direction of kinetic friction:

$$a_1 = g\sin\theta + \mu g\cos\theta,$$
$$a_2 = g\sin\theta - \mu g\cos\theta.$$

Dividing these equations, one can compute the ratio between the accelerations. It follows that $\mu\sqrt{3} = 1/2$.

Answer 23. (B) fall a bit if $\rho > \rho_0$.

Hint. Let the total mass of the boat (including yourself) be M and let the mass of the object be m. In the initial case Archimedes' principle tells us that the volume of water displaced is $V_1 = (m + M)/\rho_0$. In the final case, the volume of water displaced is $V_2 = M/\rho_0 + m/\rho$. Therefore, $V_2 - V_1 = (m/\rho - m/\rho_0) < 0$ if $\rho > \rho_0$.

Answer 24. (C)

Answer 25. (B)

Hint. The maximum density of water occurs at around 4° C.

Answer 26. (B) 34

Hint. The maximum distance for brick ± 1 (left/right) on top to remain on brick ± 2 (left/right) is reached when the center of mass of brick ± 1 is directly over the edge of brick ± 2. Thus, brick ± 1 overhangs brick ± 2 by $x_1 = d/2$, and so on. The general formula for the total distance spanned by the total brick number, $2n+2$, is

$$2x_1 + 2x_2 + 2x_3 + \cdots + 2x_n = (d/1) + (d/2) + (d/3) + \cdots + (d/n) = d \sum_{i=1}^{i=n} 1/i \geq \ell,$$

where number 2 in $2n+2$ involves two bricks as the base on each side. Therefore, $\sum_{i=1}^{i=n} 3/(10i) \geq 1$, that is already fulfilled for $n = 16$, so that the total number is 34.

Answer 27. (E) $R_{ab} > R_{ef}$

Hint. By symmetry, there are two classes of equivalent electric resistances: $R_{\mathrm{adj}} = 5R/12$ and $R_{\mathrm{opp}} = 6R/12$, when the voltage is applied between adjacent and opposite nodes, respectively. R stands for the edge electric resistance.

Answer 28. (A) $\tau_1^{(a)} < \tau_2^{(a)}$ and $\tau_2^{(b)} < \tau_1^{(b)}$

Hint. Near the surface of the water, the cold copper ball cools the warmer water around it. This water is denser than the rest of the water, so it sinks, and a convective flow is formed, which accelerates heat exchange. If the surrounding water is colder than the copper ball, the water will gradually heat up and its density will decrease, but since the ball is close to the water surface, this will not initiate a convective flow, and the heat exchange will be slower. Accordingly, $\tau_1^{(a)} < \tau_2^{(a)}$. Furthermore, the warmer copper ball at the bottom of the vessel heats up the surrounding water, its density decreases near the ball. Water with a lower density rises and colder water flows in its place. The heat exchange will be faster due to the convective flow. Accordingly, $\tau_2^{(b)} < \tau_1^{(b)}$.

Answer 29. (B)

Answer 30. (E) $P = 2^{7/2} \times P_0$.

Hint. There are two mass densities: one related to the average mass density of helicopter, ρ_{helic} and the other related to the mass density of air, ρ_{air}; as well as, two lengths: the general length of the main body, ℓ_{body}, determining how strong the force of gravity is and the length of the propellers, ℓ_{blade}, telling how strong the lift force is. More formally,

$$\rho_{\mathrm{helic}} = \alpha_1 \rho, \rho_{\mathrm{air}} = \alpha_2 \rho \text{ and } \ell_{\mathrm{body}} = \beta_1 \ell, \ell_{\mathrm{blade}} = \beta_2 \ell,$$

where $\alpha_{1,2}$ and $\beta_{1,2}$ are dimensionless scaling constants. Alongside the acceleration due to gravity g, these parameters have dimensions of $[\ell] = \mathrm{m}$, $[\rho] = \mathrm{kg/m^3}$, and $[g] = \mathrm{m/s^2}$. Units of P in terms of the SI base units: kg, m, and s, can be derived with some basic equations such that

$$[P] = \mathrm{kg} \cdot \mathrm{m^2 s^{-3}} \Rightarrow P \sim \ell^x \rho^y g^z,$$

to ultimately lead to $x = 7/2$, $y = 1$, and $z = 3/2$. Therefore, doubling all the linear dimensions implies an increase in the power output by a factor of $2^{7/2}$. This is exactly the same result that can be obtained with rather rigorous methods, other than dimensional analysis.

Answer 31. (E)

Hint. The electric fields point out along the directions in which the potential energy function decreases. Mathematically speaking, the electric field vector boils down to the negative gradient of the electric potential.

Answer 32. (B) $\mathscr{I}_1$ is real.

Hint. The two images are formed by light rays reflected on the two surfaces of the magnifying glass. The upper surface of the lens can be considered a convex mirror, and the image it creates is a virtual upright image under the lens. The rays that create the other image first pass through the lens, then get reflected by the lower surface, as a concave mirror from the point of view of imaging, and then pass through the magnifying glass again. The resulting image is an inverted real image above the lens. The photo does not show which image is located where, but with the calculation one can decide what image is real/virtual. Let the radius of the surfaces be R, the focal length of the lens f, and let us assume that the rays involved in the two mappings form a small angle with the optical axis of the magnifier. Mark the distance of the chandelier from the lens with p, and the two image distances with $|\ell_1| > 0$ and $\ell_2 > 0$. In the case of the virtual image (one sole reflection):

$$\frac{1}{p} - \frac{1}{|\ell_1|} = -\frac{2}{R}.$$

In the case of the real image, one must use the fact that the diopters (reciprocals of the focal lengths) of the imaging devices along the path of the light add up, and that the light passes through the glass twice.

$$\frac{1}{p} + \frac{1}{\ell_2} = \frac{2}{R} + \frac{2}{f}.$$

Adding and rearranging the two equations:

$$|\ell_1| - \ell_2 = |\ell_1| \times \ell_2 \times \left(\frac{2}{f} - \frac{2}{p}\right).$$

The distance of the chandelier is much greater than the focal length of the magnifier, i.e. $p \gg f$, then $|\ell_1| > \ell_2$. The sizes of the corresponding two images are also proportional to their distances.